

W, E, L, C, O, M, E,

While we wait,
please type your
name and school
in the chat.

IES PRACTICE GUIDE

WHAT WORKS CLEARINGHOUSE

Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools



NCEE 2009-4060
U.S. DEPARTMENT OF EDUCATION

ies NATIONAL CENTER FOR
EDUCATION EVALUATION
AND REGIONAL ASSISTANCE
Institute of Education Sciences



ASSISTING STUDENTS STRUGGLING WITH MATHEMATICS:

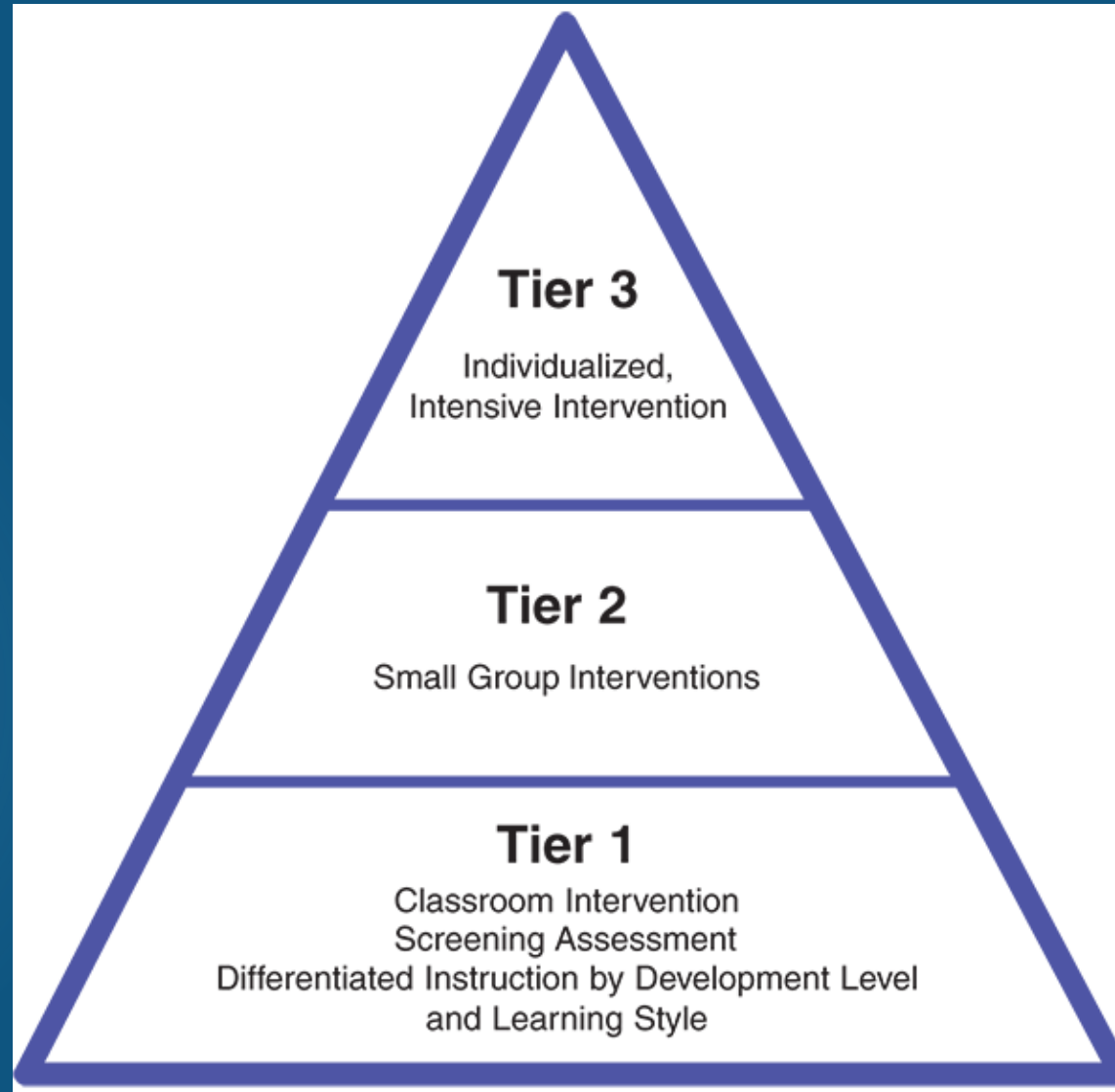
RESPONSE TO INTERVENTION (RTI)

Objectives:

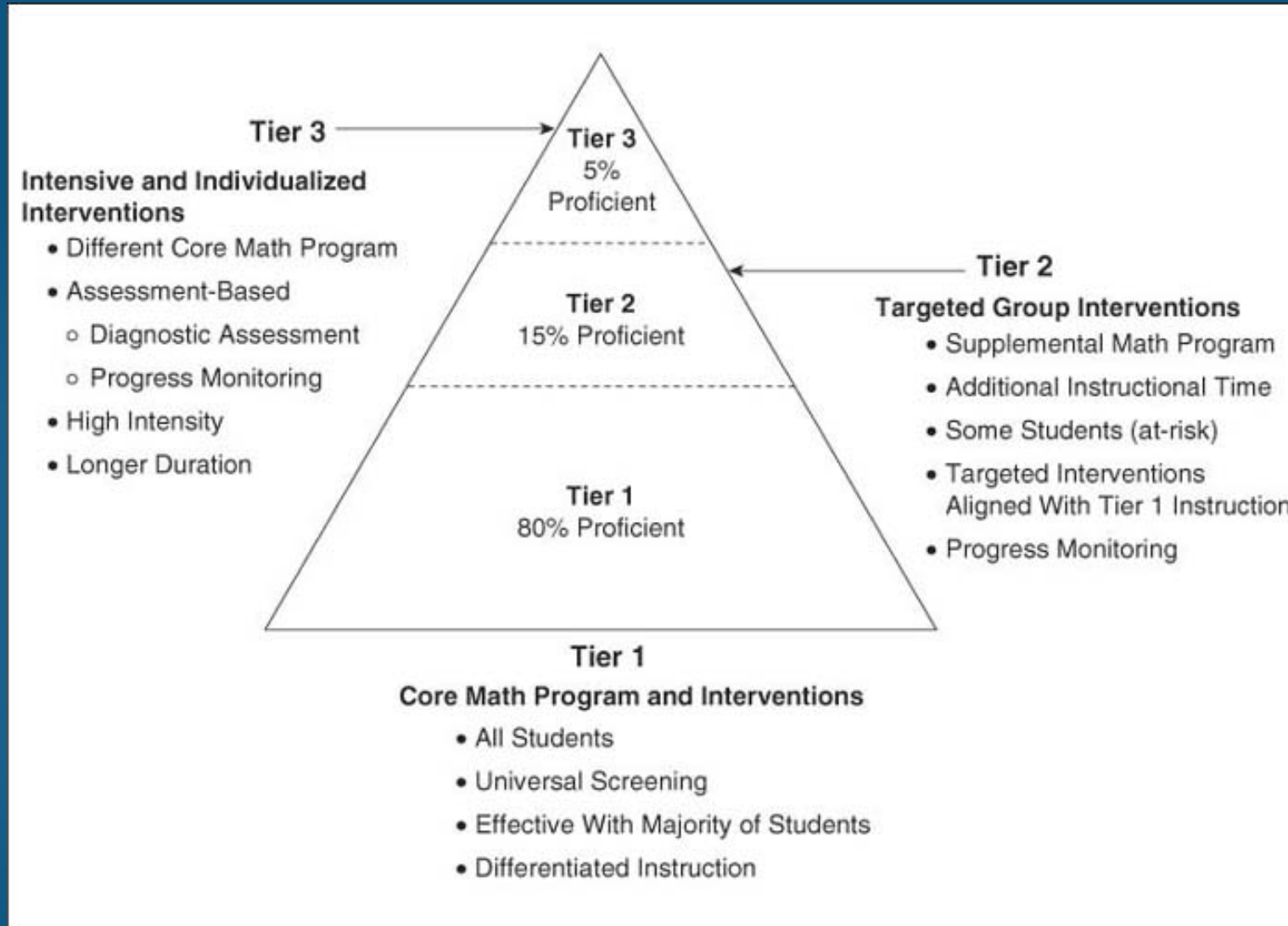
- ▶ 1. To identify and support students struggling in math through eight recommendations.
- ▶ 2. To implement recommendations within an RTI framework.

What is RTI?

Response to Intervention



- ▶ Response to Intervention (**RTI**) is a multi-tier **approach** to the early identification and support of students with learning and behavior needs. The **RTI** process begins with high-quality instruction and universal screening of all children in the general education classroom.



Tier 1 Interventions

- The key component of tiered instruction, all students receive instruction within an evidence-based, scientifically researched core program.
- Is synonymous with the core reading or math curriculum that is typically aligned with state standards.
- All children receive Tier 1 instruction
- Assessed every 6-8 weeks.

Tier 2 Interventions

- Need to spend 25%+ time on these interventions versus Tier 1.
- Focusing on both core and supplemental topics.
- These interventions need to be assessed at least every week.
- Consists of children who fall below the expected levels of accomplishment (called benchmarks) and are at some risk for academic failure but who are still above levels considered to indicate a high risk for failure.
- The needs of these students are identified through the assessment process, and instructional programs are delivered that focus on their specific needs.

Tier 3 Interventions

- Tier 3 consists of children who are considered to be at high risk for failure and, if not responsive, are considered to be candidates for identification as having special education needs.
- The groups of students at Tier 3 are of much smaller sizes, ranging from 3 to 5 children, with some models using one-to-one instruction.
- Students should be assessed every 1-2 days.
- Intensive individualized interventions.

Recommendation 1 – Screen and monitor progress

Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

- ▶ Conduct universal screening
- ▶ Monitor student progress
- ▶ Regroup students based on monitoring data



- ▶ Universal screening is an assessment that helps teachers identify students who are at risk of not meeting grade level learning goals.
- ▶ Screenings provide valuable information to teachers.

What Screenings do we use?

- ▶ Results of Previous year STAAR Score- Could be used as the First stage screening
- ▶ Benchmark
- ▶ Formative Tests



Screening assessments in the classroom are like health screenings at a doctor's office.

They check for warning signs to see if you might be at risk. For example, if you can't read the chart then the doctor is likely to give you contact lenses or glasses so that you can see more clearly. If you can read the chart, then the doctor will send you on your way until your next eye screening.



Recommendation 2 – Teach the foundations of arithmetic

Instructional materials for students receiving interventions should focus intensely on in-depth treatment of **whole numbers** in kindergarten through 5th grade and on **rational numbers** in grades 4 through 8. These materials should be selected by committee.

- ▶ Whole numbers (grades K-5)
- ▶ Rational numbers (grades 4-8)



K-5 Focus on Whole Numbers

- ▶ Lines of reasoning
- ▶ Calculation methods
- ▶ Basic counting numbers 0, 1, 2, 3, 4, 5, 6, ... and so on.
- ▶ Counting; counting up
- ▶ Place Value- base 10 system; concrete and visual representations
- ▶ Number paths and number lines
- ▶ Composition and decomposition; to understand Place Value and multi-digit operations
- ▶ Operations: Addition, Subtraction Multiplication and Division (the reasoning that underlies algorithms)
- ▶ Interventions should include materials to build fluent retrieval of basic math facts



4-5 Focus on Rational Numbers

- ▶ A rational number is a number that can be expressed as a fraction where both the numerator and the denominator in the fraction are integers. The denominator in a rational number cannot be zero.
- ▶ Fractions and Decimals
- ▶ Decimals and percentages
- ▶ Lines of reasoning
- ▶ Calculation methods

Let's Process



HOW MANY OF YOU CAN SWIM?

- Can you float? Can you swim underwater?
- How's your kicking technique, your breathing, your speed, your stamina?
- These will definitely define your swimming capabilities; however, this is not the case for some of our students when it comes to Math.
- Some of them have never been able to float, but you're a great coach.
- If a swimmer can swim and get to the end line, that's success. Our students' success begins on how they see themselves.

NICK VUJICIC
"I was never crippled
until I lost hope."



Recommendation 3

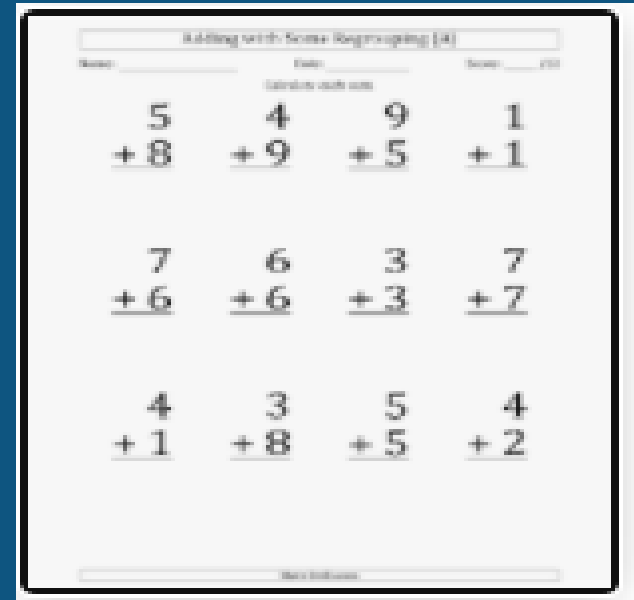
Instruction during the intervention should be **explicit and systematic**. This includes providing models of proficient problems solving, verbalization of through processes, guided practice, corrective feedback, and frequent cumulative review.

- ▶ Explicit and systematic
 - Modeling
 - Scaffolded practice
 - Corrective feedback
 - Cumulative review



Strategy

VS



Drill and Practice



Strategy

- ▶ Instruction should gradually build proficiency by introducing concepts in logical order and provide examples of each.
 - Ex. Teach place value before teaching addition and subtraction with regrouping.
 - Make their thinking process audible as they model the steps
- ▶ Provide opportunities to solve problems to communicate the strategy
 - During guided practice, teacher transfers work to students
 - Explicit instruction begins with the teacher and students solve problems together
 - Teacher clarifies and suggests improvements
- ▶ Check for understanding

Recommendation 4 – Word Problems

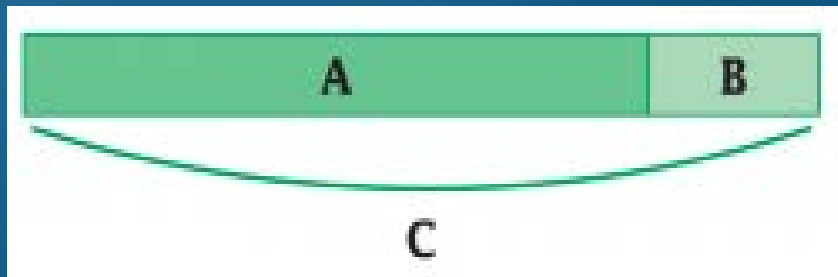
Interventions should include instruction on solving word problems that is based on common underlying structures.

- ▶ Teaching students to decide which operation applies to which type of story problems
- ▶ Teaching students to recognize common structures between familiar and unfamiliar problems and to transfer known solution method from familiar to unfamiliar problems
- ▶ Difficulties caused by the simplified approach of teaching keywords to identify operations.

Various problem types

"Change" Problems

- Change problems describe situations in which a quantity (such as children or pencils) is either increased or decreased.
- Students determine whether to add or subtract by determining whether the change in the quantity is more or less.



"Compare" Problems

- This focus on comparing two different types of items in two different sets (pears and apples, boys and girls, hot and cold items).
- Students add or subtract by determining whether they need to calculate the unknown difference (subtract), unknown compared amount (add), or unknown referent amount (subtract).

There are 21 hamsters and 32 kittens at the pet store. How many more kittens are at the pet store than hamsters?



How to carry out this recommendation

Include numerous clear models of easy and difficult problems, with accompanying teacher think-alouds.



Provide students opportunities to solve problems in a group and communicate problem-solving strategies.

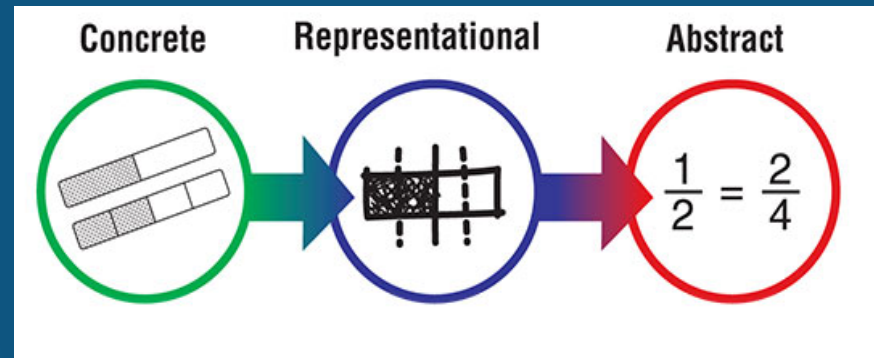


Include cumulative review in each session.



Recommendation 5 - Visual representations

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

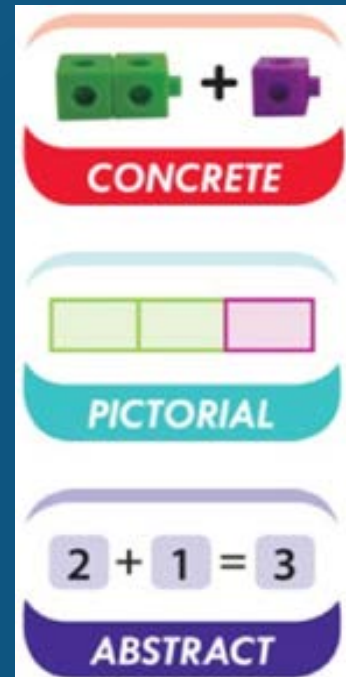


CRA

Concrete- Hands-on experiences with manipulatives when introducing a new concept or skill.

Representation- *Modeling their thinking*. Students can visualize the concept and represent it pictorially through models. Mental images assist students in dissecting challenging word problems and developing logical thinking.

Abstract- Students use abstract numbers and symbols when they understand what they mean.



Students struggling with facts-

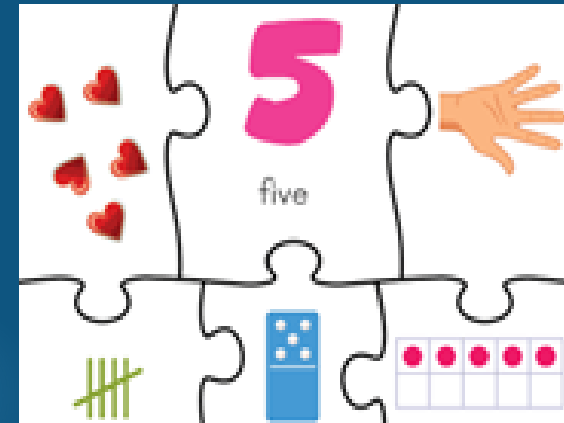
Piece they are usually missing is Spatial Relationships



Four Number relationships that make up number sense

- ▶ Spatial relationship- having a visual picture of a number
- ▶ One more and one less, Two more and two less
- ▶ Understanding benchmarks of five and ten
- ▶ Part/Part, whole

Tools that can help build that: Ten frames, Number bonds, Reckenrek, Math Rack



P₃ O₁ S₁ I₁ T₇ I₁ V₄ E₁

S₁ A₁ F₄ E₁

E₁ N₁ C₃ O₁ U₁ R₁ A₁ G₂ E₁

Algorithm with Place Value Chart

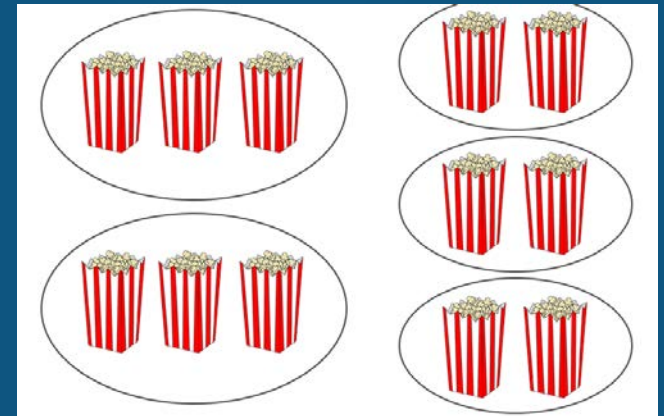
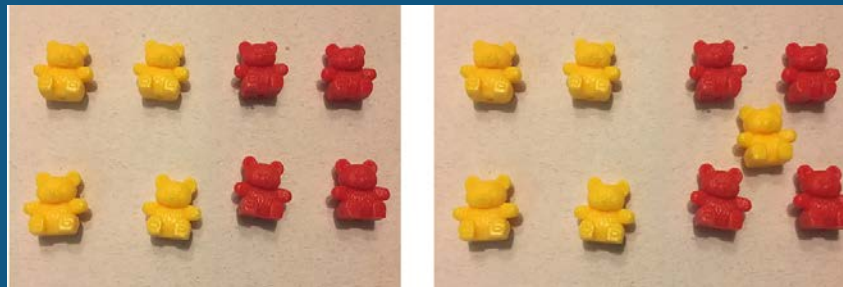
$$26 + 35 = \underline{\quad}$$

	Tens	Ones
26		●●●●
+ 35		●●●●

(Note: A green bracket in the original image groups the 10 dots in the Ones column, indicating a carry to the Tens column.)

SAME BUT DIFFERENT MATH

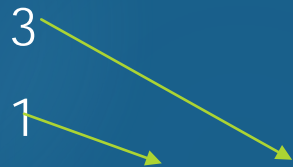
- ▶ The root of helping kids develop math **fact fluency** is through the connections and relationships of numbers that we help kids develop. The more connections and relationships around numbers that they build, the more fluent kids become with their facts.



Solve: $24 + 68 + 97 =$

- ▶ Flexibility
- ▶ Discussion

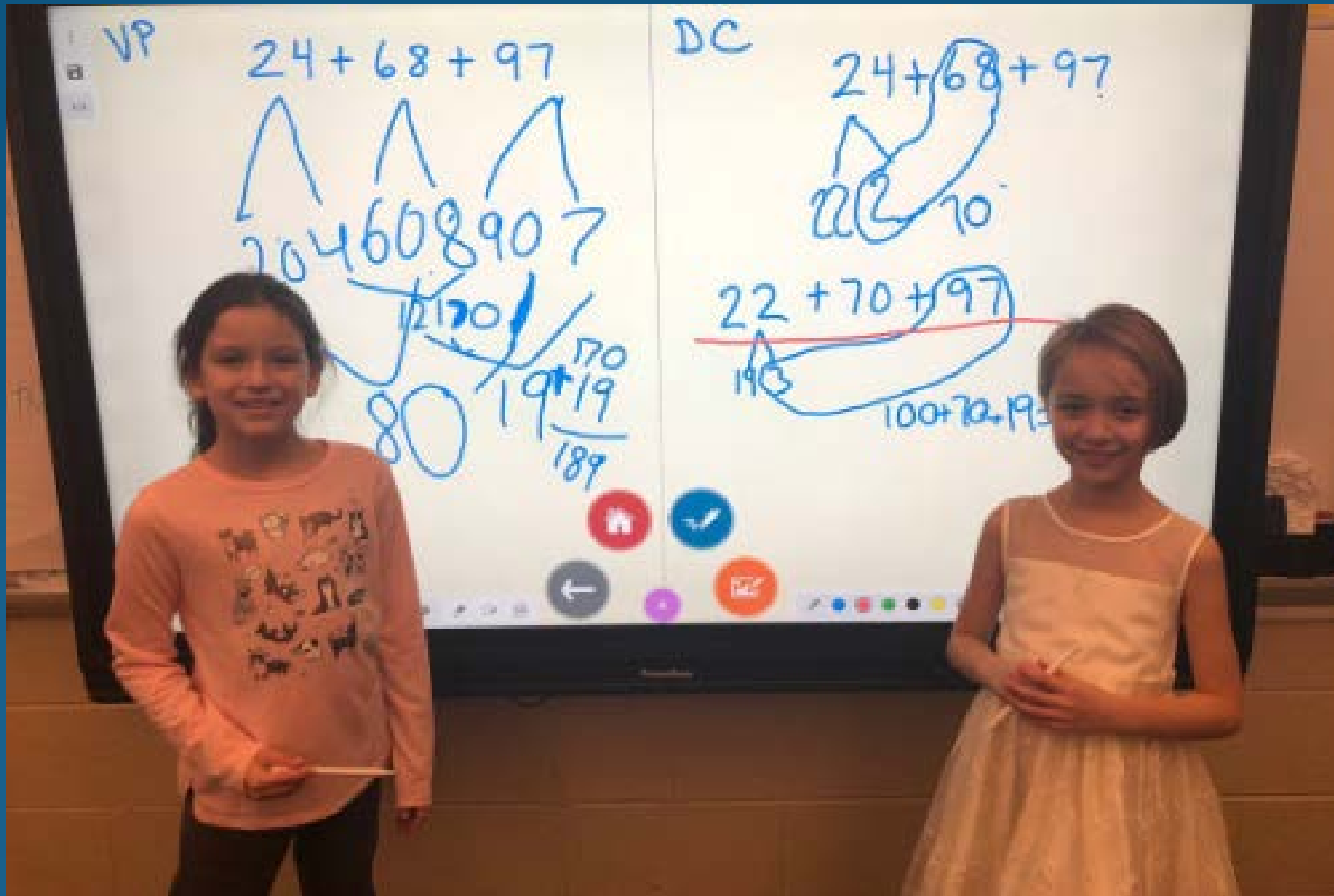
$$24 + 68 + 97$$



$$20 + 69 + 100$$

$$89 + 100 = 189$$

$$\begin{array}{r} 24 \\ 26 \\ \hline +97 \end{array}$$



Are we focused on answer getting or problem solving?

9

9

7

$9 + 7$

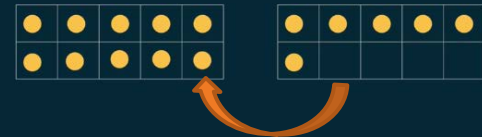
$9 + 7$

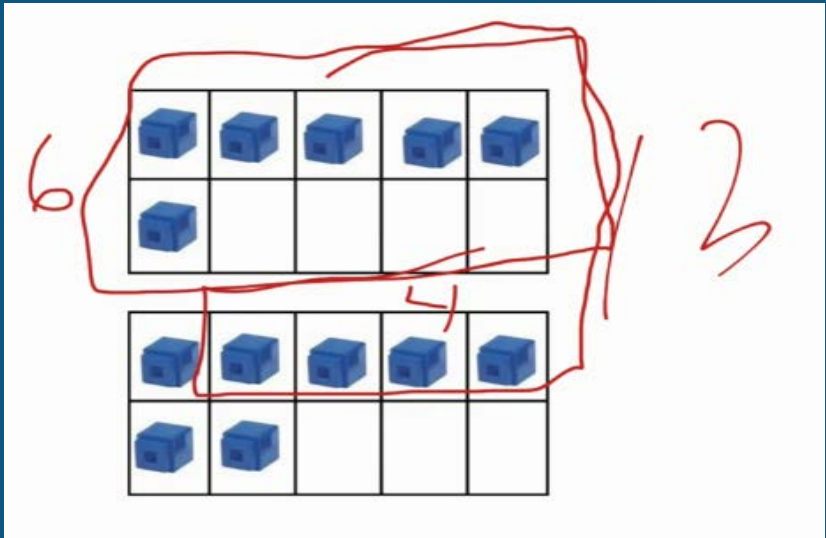
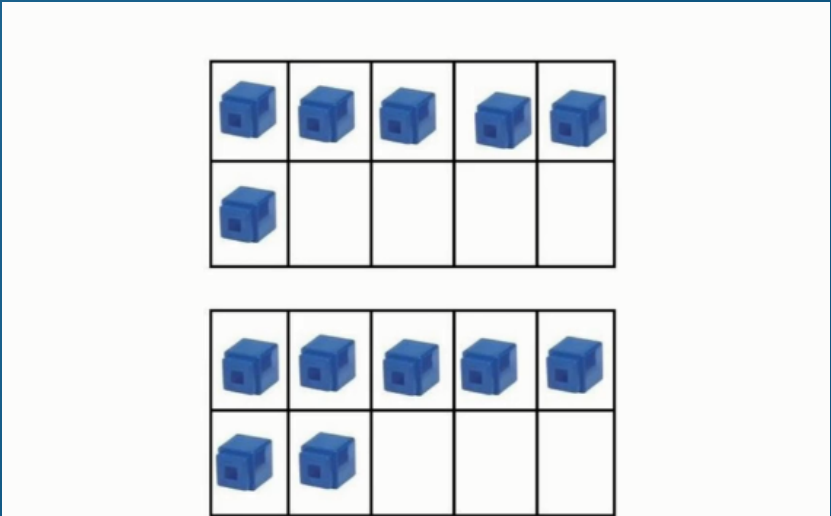
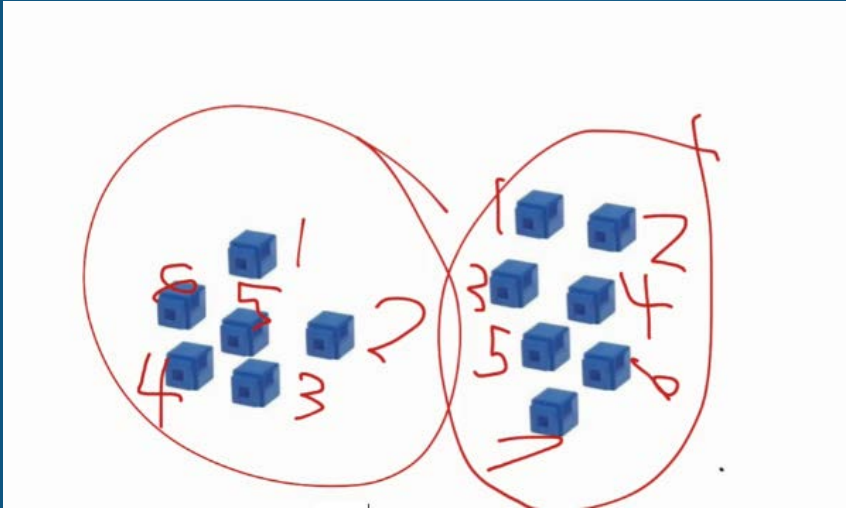
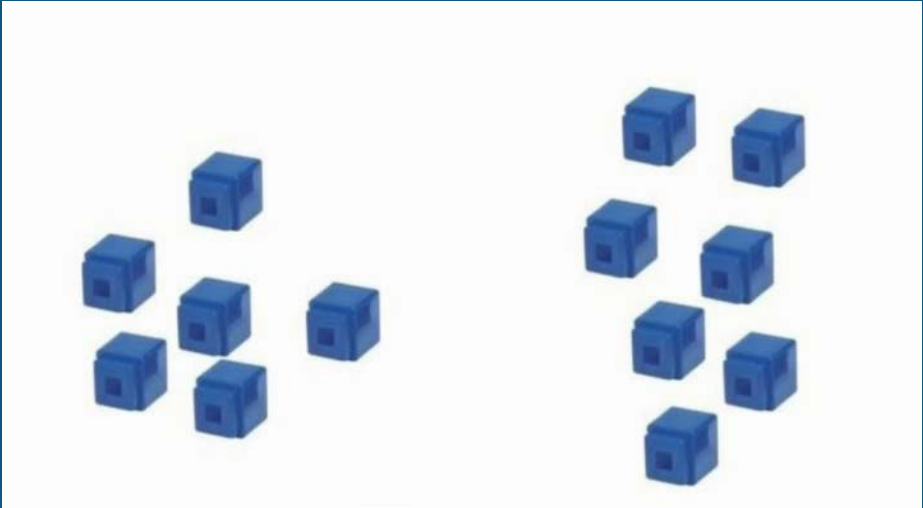
$10 + 6$

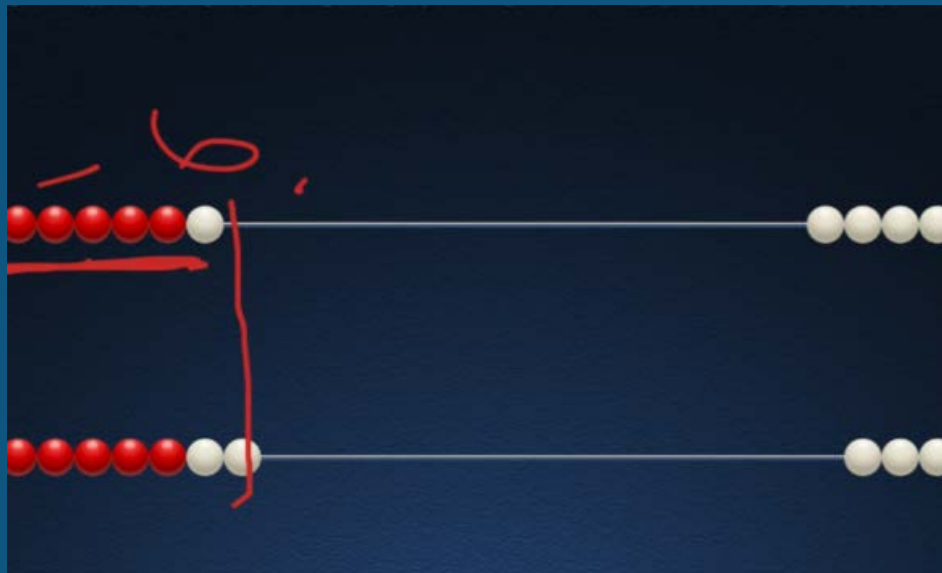
$9 + 7$



$9 + 7$

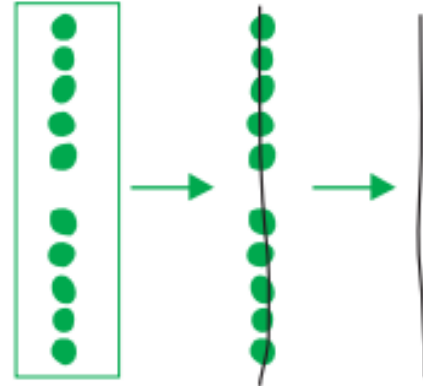




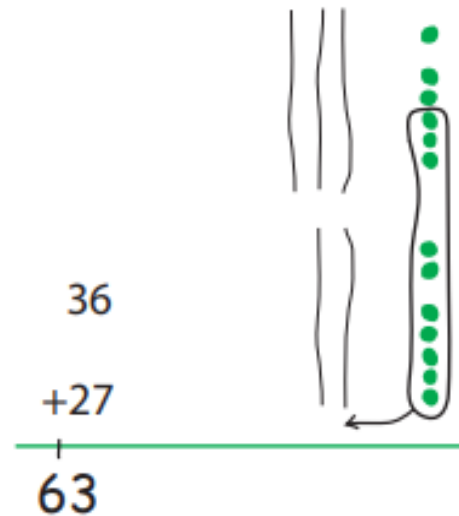


Using visual representations for multidigit addition.

A group of ten can be drawn with a long line to indicate that ten ones are joined to form one ten:

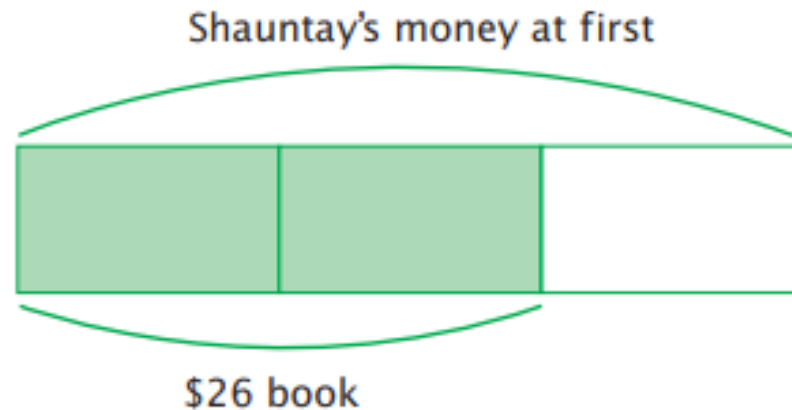


Simple drawings help make sense of two-digit addition with regrouping:



Strip diagrams can help students make sense of fractions.

Shauntay spent $\frac{2}{3}$ of the money she had on a book that cost \$26. How much money did Shauntay have before she bought the book?



2 parts \longrightarrow \$26
1 part \longrightarrow $\$26 \div 2 = \13
3 parts \longrightarrow $3 \times \$13 = \39

Shauntay's had \$39

27 things are divided equally among 3 groups.

$$27 \div 3 = ?$$

$$3 \times ? = 27$$

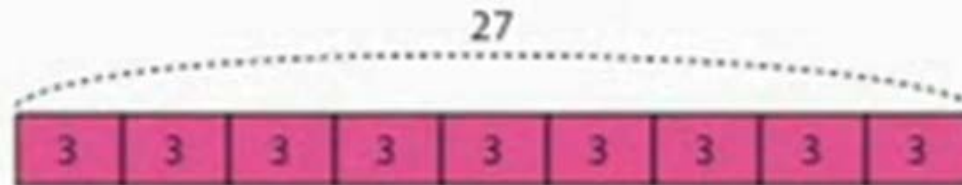


How many things are in each group?

27 things are divided into groups of 3.

$$27 \div 3 = ?$$

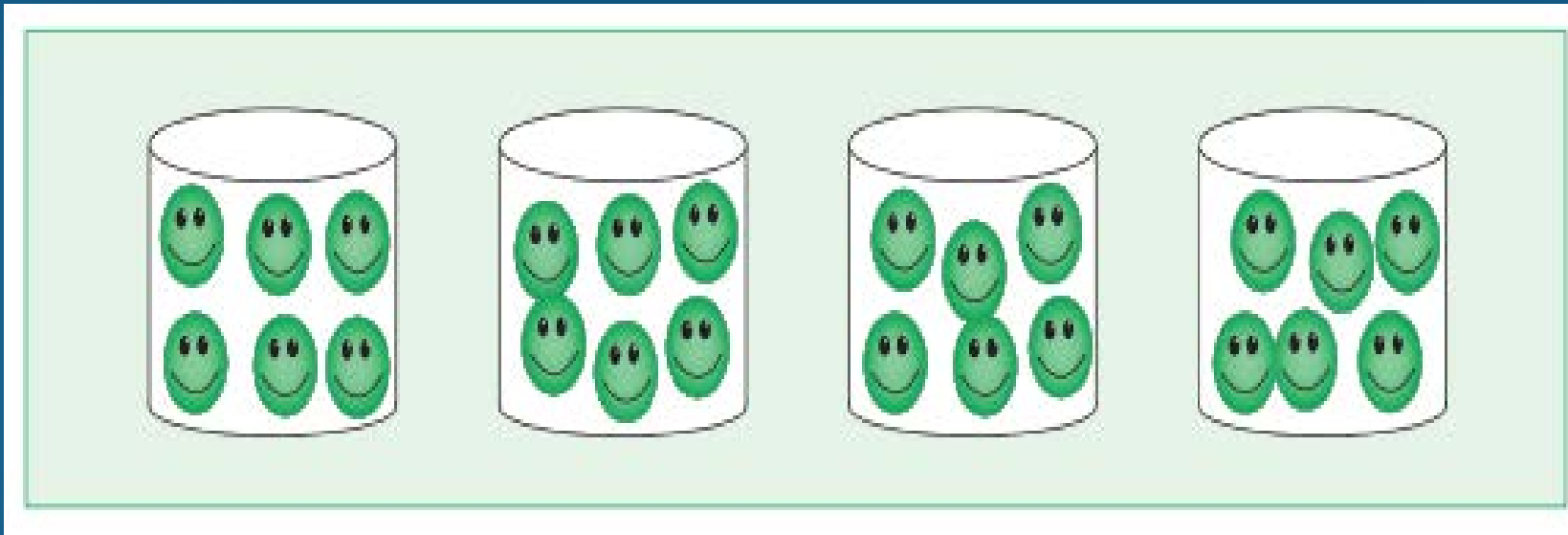
$$? \times 3 = 27$$



How many groups are there?

Manipulatives can help students understand that four multiplied by six means four groups of six, which means 24 total objects.

$$4 \times 6 = 24$$



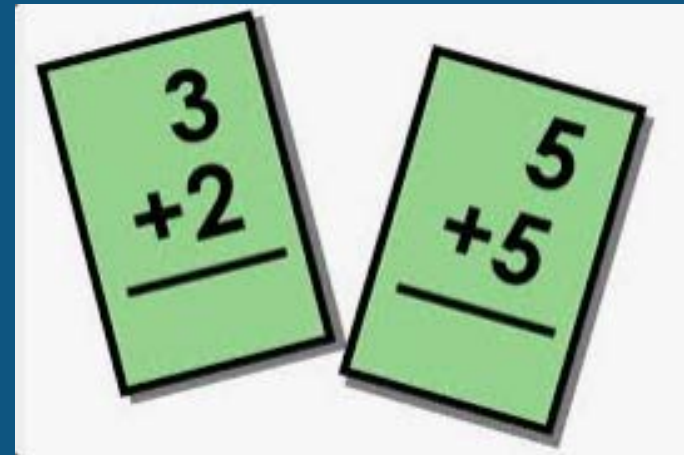
Recommendation 6 – Fluent retrieval of facts

Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

- Quick retrieval of basic arithmetic facts is critical for success in mathematics.
- Weak ability to retrieve math facts is likely to impede understanding of concepts students encounter with rational numbers.
- It is recommended to take 5 – 10 minutes to build proficiency.

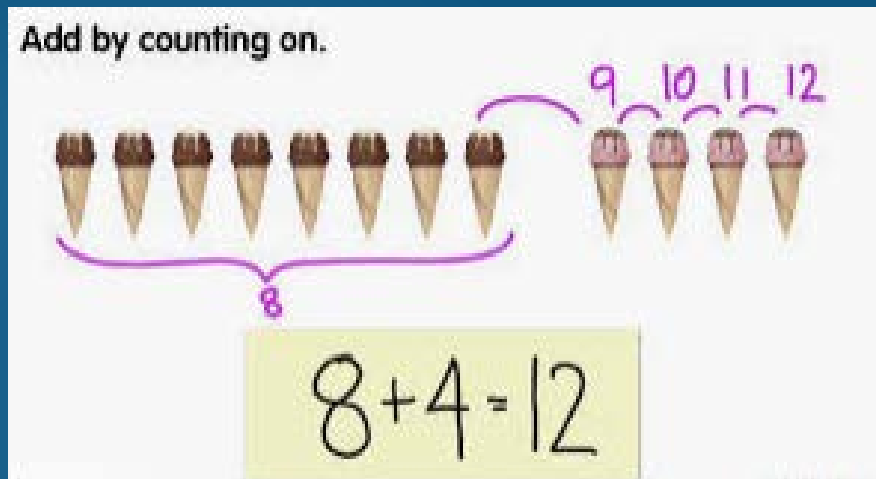
Building quick retrieval of basic facts

- It is recommended to start in 2nd grade.
- The goal is quick retrieval of facts using digits 0-9 without pencil and paper or manipulatives.
- Presenting facts in number families (such as $7 \times 8 = 56$, $8 \times 7 = 56$, $56/7 = 8$, and $56/8 = 7$) shows promise for improving student fluency.



Explicitly teach strategies to K-2 for efficient counting

- Counting up strategy
- Not only will it improve fact fluency but students will understand commutative property.



For grades 2-8, teach students how to use the knowledge of properties.

- The use of composing and decomposing, and applying the distributive property to situations can increasingly allow students to learn how to quickly retrieve facts.

Compose
to combine
4 tens → 40
 $1,000 + 200 + 40 + 7 = 1,247$
combine amounts to make another

Decompose
to break apart

1,247

1,000 + 200 + 40 + 7

Properties of Multiplication

Commutative Property of multiplication

Two numbers can be multiplied in any order and the product (answer) will be the same.

Switch it up!

$$3 \times 5 = 15$$
$$5 \times 3 = 15$$

Properties of Multiplication

Associative Property of multiplication

You can multiply 3 or more numbers. If you change the grouping of the factors, the product (answer) will stay the same.

Change the groups man!

$$(2 \times 5) \times 3 = 30$$
$$2 \times (5 \times 3) = 30$$

Properties of Multiplication

Distributive Property of multiplication

A multiplication fact can be broken apart into the sum of two other multiplication facts.

Break it up!

$$3 \times 6 = (2 \times 6) + (1 \times 6)$$
$$12 + 6 = 18$$

So... $3 \times 6 = 18$

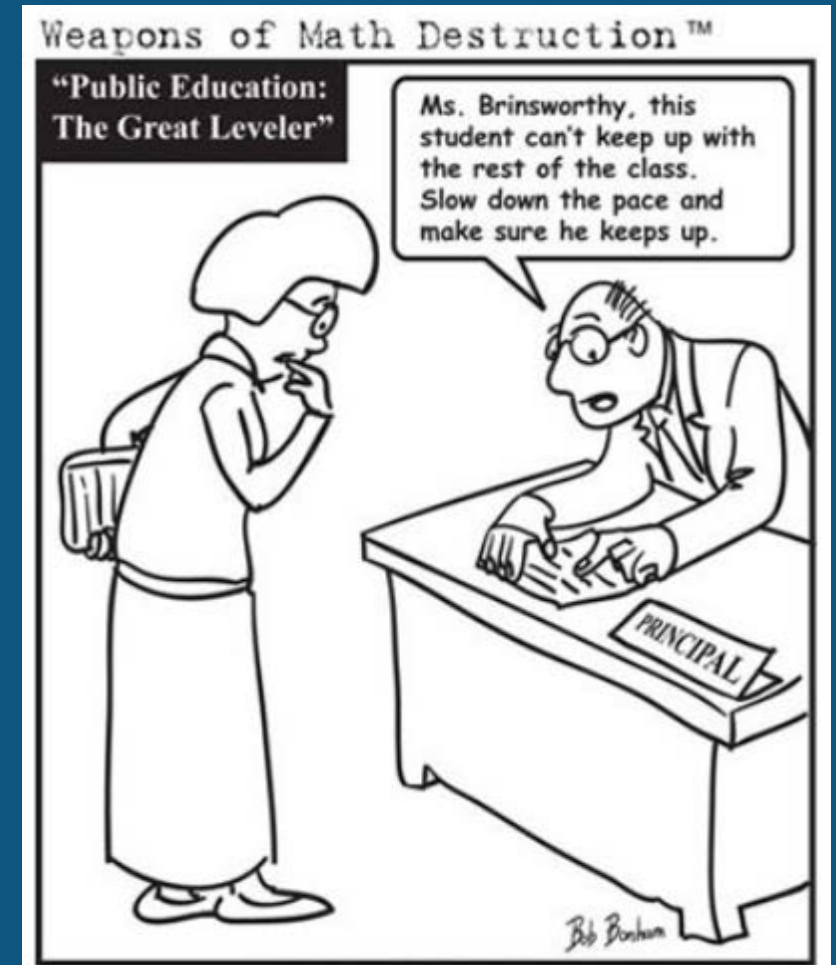
Recommendation 7 – Monitor Progress

Monitor the progress of students receiving supplemental instruction and other students who are at risk.

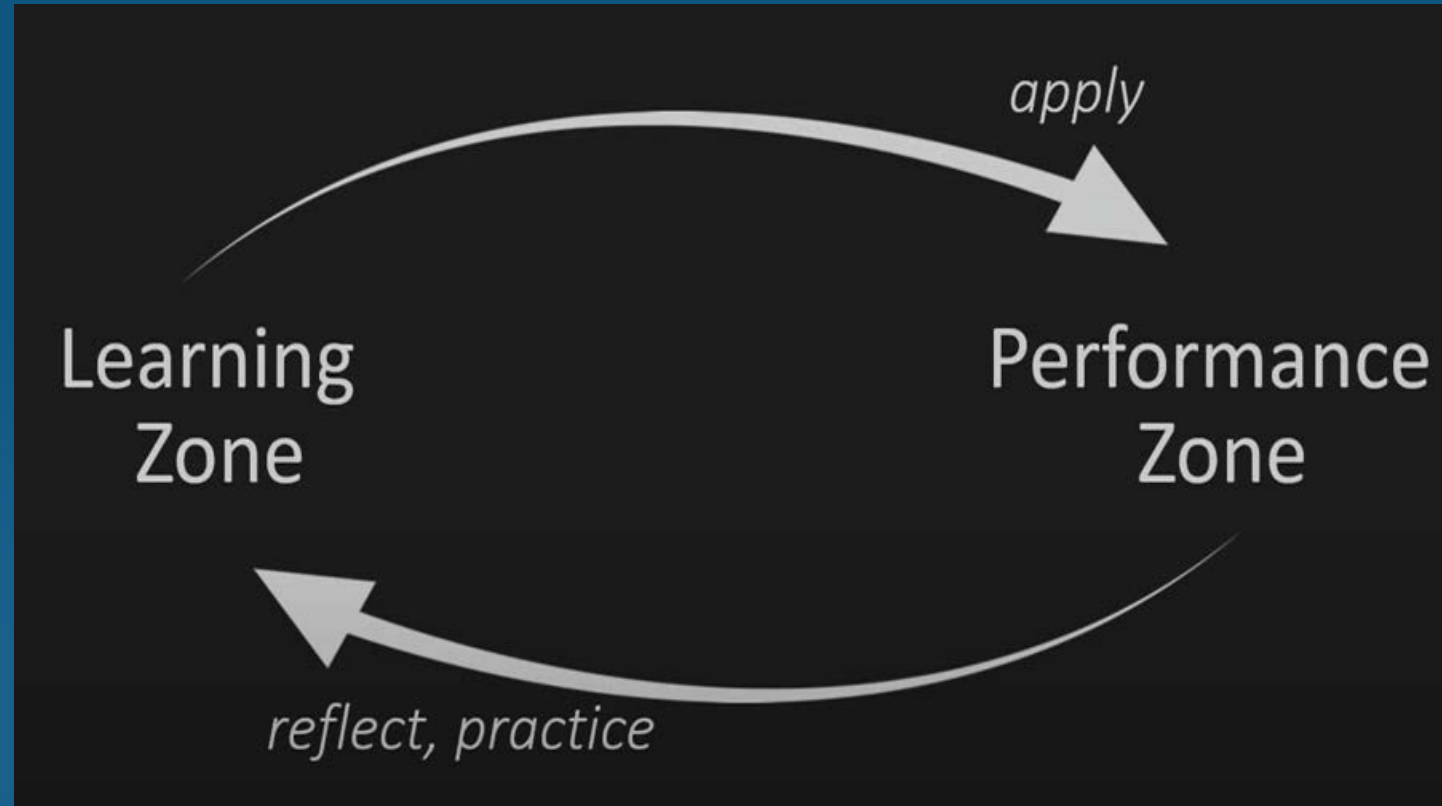
- Monitor the progress of tier 2, tier 3, and borderline tier 1 students at least once a month.
- Use curriculum-embedded assessments in interventions to determine whether students are learning from the intervention.
- Use progress monitoring data to regroup students when necessary.



- Adjust intervention as needed based on data
 - Frequency
 - Intensity
 - Duration
- Develop new intervention if needed.



D₂ A₁ T₁ A₁



C H A N G E

Recommendation 8

CREATING POSITIVE CLASSROOM CULTURE



Include **motivational** strategies in tier 2 and tier 3 interventions.



----Strategy

- Reinforce or praise students for their effort and for attending to and being engaged in the lesson.
- Consider rewarding student accomplishments.
- Allow students to chart their progress and to set goals for improvement.

quarter: _ math @ @ + @

15																				
14																				
13																				
12																				
11																				
10																				
9																				
8																				
7																				
6																				
5																				
4																				
3																				
2																				
1																				
Date																				

Like a math equation, in order to get the most, you have to know how to convert negatives into positives.



Cover-Copy-Compare Strategy

Cover-Copy-Compare:

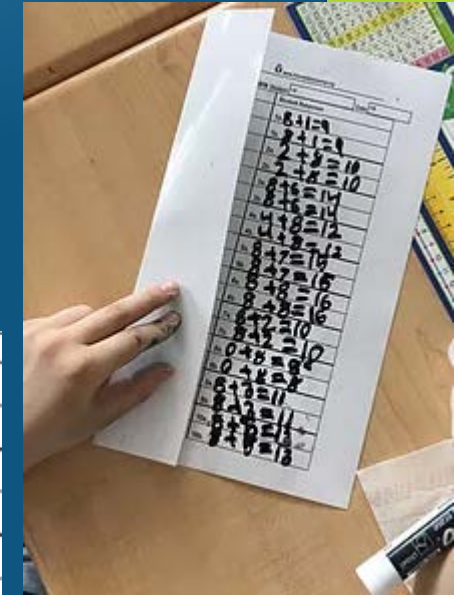
Math Computational Fluency-Building Intervention

The student is given sheet with correctly completed math problems in left column and index card.

For each problem, the student:

- studies the model
- covers the model with index card
- copies the problem from memory
- solves the problem
- uncovers the correctly completed model to check answer

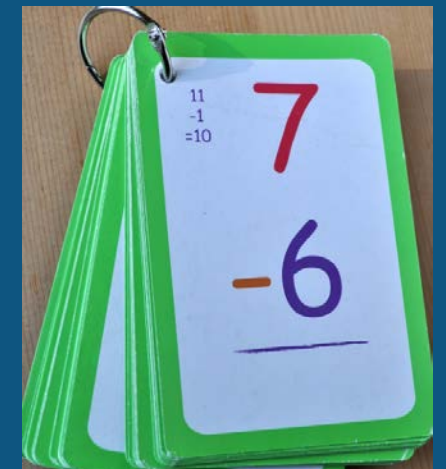
Math Facts	Student Response
1. $9 \times 7 = 63$	1a. $9 \times 7 = 63$
	1b.
2. $9 \times 2 = 18$	2a.
	2b.
3. $9 \times 4 = 36$	3a.
	3b.
4. $9 \times 1 = 9$	4a.
	4b.
5. $9 \times 9 = 81$	5a.
	5b.
6. $9 \times 6 = 54$	6a.
	6b.
7. $9 \times 3 = 27$	7a.
	7b.
8. $9 \times 5 = 45$	8a.
	8b.
9. $9 \times 10 = 90$	9a.
	9b.
10. $9 \times 8 = 72$	10a.



Strategies for fluency

► Flash Cards Uno Cards or Deck of playing cards

- **Around the world**-Have students sit in a circle. Ask one student to stand behind a child in the in the circle. State a math problem aloud or display a flash card with a math problem on it, and ask both students to figure out the answer. The first student who calls out the correct answer moves to stand behind the next child in the circle. The object of the game is to see how far one student can "travel" without making any mistakes.
- **War**- Each person has cards from the deck. Two students play against each other. The first student pulls two cards out and adds the numbers. The second student does the same thing. Student with the highest amount keeps the cards. Or with subtraction the student that has the lowest number keeps the cards.
- **Transition** time- Keep cards on a ring and have them solve as they wait.



----Strategy

- ▶ **Musical math-** whole group activity provide the class with facts in a clear sheet protector. They start solving when the music begins. When the music stops they pass their card to the person sitting next to them. The class continues until all the cards are filled. **Taking the anxiety out of timed tests.** At the end, the whole class reviews the problems and corrects problems that are wrong.

<https://www.didax.com/math/virtual-manipulatives.html>

**YOU CAN TEACH A
STUDENT A LESSON
FOR A DAY; BUT IF
YOU CAN TEACH HIM
TO LEARN BY
CREATING CURIOSITY,
HE WILL CONTINUE
THE LEARNING
PROCESS AS LONG AS
HE LIVES.**

CLAY P. BEDFORD

